Exercise 8B: Solutions

1 **a** $x = 73^\circ$ (alternate segments)
$y = 81^\circ$ (alternate segments)

b $\angle T = 90^\circ$
$\therefore x = 90^\circ - 33^\circ = 57^\circ$
$q = 57^\circ$ (alternate segment theorem)

c $y = 74^\circ$ (alternate segments)
$$z = \frac{180^\circ - 74^\circ}{2} = 53^\circ$$
$x = 53^\circ$ (alternate segments)

d $x = 180^\circ - 80^\circ - 40^\circ = 60^\circ$
Use the alternate segment theorem to find the other angles.
$y = 180^\circ - 60^\circ - 60^\circ = 60^\circ$
$w = 180^\circ - 40^\circ - 40^\circ = 100^\circ$
$z = 180^\circ - 80^\circ - 80^\circ = 20^\circ$

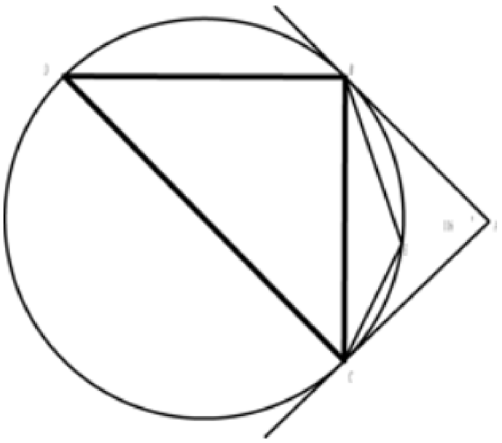
e $w = z = x = 54^\circ$ (alternate segment, alternate angles and isosceles triangle PTS)
$y = 180^\circ - 54^\circ - 54^\circ = 72^\circ$

2 **a** $\angle BCX = 40^\circ$

b $\angle CBD = 40^\circ$

c $\angle ABC = 2 \times 40^\circ = 80^\circ$

3



Triangle ABC is isosceles;
$$\angle ABC = \angle ACB = \frac{180^\circ - 116^\circ}{2} = 32^\circ$$
Using the alternate angle theorem,
$\angle BDC = \angle ACB = 32^\circ$.
$\angle BEC + \angle BDC = 180^\circ$
(opposite angles in cyclic quadrilateral $BCED$)
$\therefore BEC = 180^\circ - 32^\circ = 148^\circ$

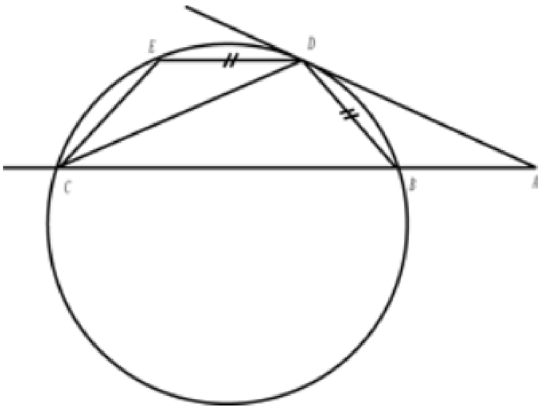
4 In $\triangle CAT$,
$\angle ACB = 180^\circ - 30^\circ - 110^\circ = 40^\circ$
The alternate segment theorem shows
$\angle BAT = 40^\circ$

$\therefore \quad \angle CAB = 110^\circ - 40^\circ = 70^\circ$

In $\triangle CAB$,

$\angle ABC = 180^\circ - 40^\circ - 70^\circ = 70^\circ$

5



There are multiple ways of proving this result.

$\angle ADB = \angle DCB$ (alternate segment)

$\angle DCB = \angle DCE$ (subtended by equal arcs)

$\therefore \angle ADB = \angle DCE$

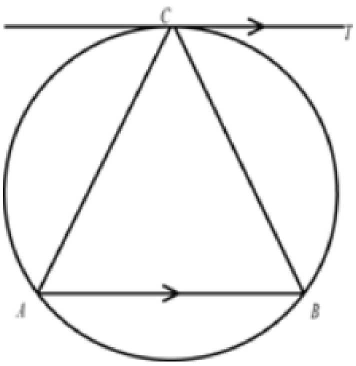
$\angle DBA + \angle DBC = 180^\circ$

$\angle DEC + \angle DBC = 180^\circ$

$\therefore \angle DBA = \angle DEC$

$\therefore$ triangles *ABD* and *CDE* are similar, since two pairs of opposite angles are equal.

6



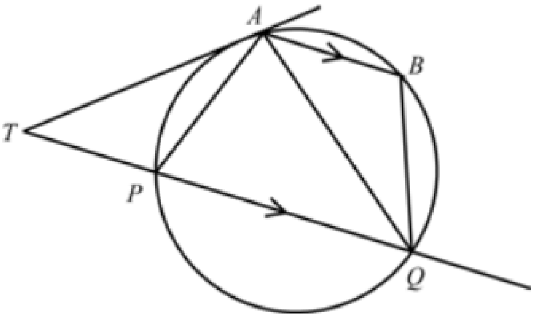
$\angle TCB = \angle CBA$ (alternate angles)

$\angle TCB = \angle CAB$ (alternate segment)

$\therefore \angle CBA = \angle CAB$

ABC is an isosceles triangle with $CA = CB$.

7



$\angle TAP = \angle AQP$ (alternate segment)

$\angle AQP = \angle BAQ$ (alternate angles)

$\therefore \angle TAP = \angle BAQ$

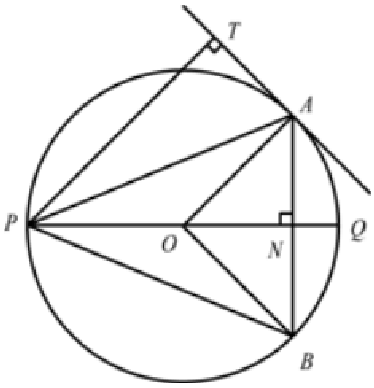
$\angle APT + \angle APQ = 180^\circ$ (adjacent angles)

$\angle AQB + \angle APQ = 180^\circ$ (opposite angles)

$\therefore \quad \angle APT = \angle ABQ$

Triangles PAT and BAQ are similar, since two pairs of opposite angles are equal.

8



Let T be the point where the perpendicular from P meets the tangent at A

Let O be the centre of the circle.

Join PA and PB.

Consider triangles OAN and OBN:

$\angle ANO = \angle BNO = 90^\circ$

$OA = OB$ (radii)

ON is common to both triangles.

$\therefore \angle AON \equiv \angle BON$ (RHS)

$AN = BN$

Now consider triangles PAN and PBN:

$AN = BN$

$\angle PNA = \angle PNB = 90^\circ$

PN is common to both triangles.

$\therefore PAN \equiv PBN$ (SAS)

$\angle PAN = \angle PBN$

Now consider triangles PAT and PAN:

$\angle PBN = \angle PAT$ (alternate segment theorem)

$\therefore \angle PAT = \angle PAN$

$\angle PTA = \angle PNA = 90^\circ$

PA is common to both triangles.

$\therefore \angle PAT \equiv \angle PAN$ (AAS)

$PT = PN$